Sampling limits and critical bandwidth for letter discrimination in peripheral vision.

Relationship between acuity for gratings and for tumbling-E letters in peripheral vision

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Earlier studies have reported that grating resolution is sampling-limited in peripheral vision but that letter acuity is generally poorer than grating acuity. These results suggest that peripheral resolution of objects with rich Fourier spectra may be limited by some factor other than neural sampling. To examine this suggestion we formulated and tested the hypothesis that letter acuity in the periphery is sampling-limited, just as it is for extended and truncated gratings. We tested this hypothesis with improved methodology to avoid the confounding factors of target similarity, alphabet size, individual variation, peripheral refractive error, and stimulus size. Acuity was measured for an orientation-discrimination task (horizontal versus vertical) for a three-bar resolution target and for a block-E letter in which all strokes have the same length. We confirmed previous reports in the literature that acuity for these targets is worse than for extended sinusoidal gratings. To account for these results quantitatively, we used difference-spectrum analysis to identify those frequency components of the targets that might form a basis for performing the visual discrimination task. We find that discrimination performance for the three-bar targets and the block-E letters can be accounted for by a sampling-limited model, provided that the limited number of cycles that are present in the characteristic frequency of the stimulus is taken into account. Quantitative differences in acuity for discriminating other letter pairs (e.g., right versus left letters E or characters with short central strokes) could not be attributed to undersampling of either the characteristic frequency or the frequency of maximum energy in the difference spectrum. These results suggest additional tests of the sampling theory of visual resolution, which are the subject of a companion paper [J. Opt. Soc. Am. A. **16**, 2334–2342 (1999)].
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1. INTRODUCTION

Several previous studies have found that spatial acuity in peripheral vision is lower for letter targets than for sinusoidal gratings.[1–4] Why should this be true? Does this difference in acuity indicate that different visual mechanisms limit performance for these two classes of visual stimuli, or might the difference be due to methodological factors? For example, perhaps an inappropriate metric was used to specify and compare the critical spatial dimensions of the two types of stimulus, one of which is spectrally simple (gratings) and the other of which is spectrally complex (letters). The usual convention is to equate the smallest detail (i.e., stroke width) of a letter with the half-period of the equivalent grating. In other words, the equivalence is based on the characteristic frequency of the letter target, which equals $1/(2 \times \text{stroke width})$. By this convention a 20/20 letter drawn with strokes subtending 1 arc min is equivalent to a 30-cycle-per-degree (c/deg) grating. This convention seems reasonable since the spectrum of the archetypal Snellen E letter includes a prominent peak of energy at the characteristic frequency of 2.5 cycles per letter (c/let), which corresponds to 30 c/deg when the letter is 5 arc min tall. However, this convention ignores the fact that letters contain a broad spectrum of spatial frequencies above and below the characteristic frequency that may be important for letter recognition. Which frequency components are critical for letter recognition in peripheral vision is at present unknown, although there is evidence that frequency components lower than the characteristic frequency may be sufficient for recognition in peripheral vision[5] as well as in central vision.[6–9] Thus it is possible that the reported differences between acuity for letters and for gratings is specious and that a better agreement might occur if a more appropriate basis for comparison could be devised.

Other methodological factors may also play a role in accounting for the reported discrepancy between acuity for letters and for gratings. Measurement of grating acuity typically involves the discrimination of a test target from a small number of alternatives (e.g., horizontal versus vertical), whereas letter acuity typically requires the identification of letters from a larger set (e.g., the Sloan letters or the entire alphanumeric alphabet). This difference would cause letter acuity to be worse than grating acuity because increasing the number of stimulus alternatives introduces uncertainty into the decision process, which is known to affect psychophysical performance adversely in a variety of visual tasks.[10] The method used to vary stimulus size may also be important. Letter acuity is measured by spatially scaling the target to find the smallest size for which letters can be identified reliably. Consequently the number of cycles of any given spatial frequency component of the letter (expressed in cycles per letter) is fixed. Grating acuity, however, is often measured by varying the spatial frequency of a patch of grating viewed through a window of fixed size, consequently the number of cycles present in the stimulus covaries with spatial frequency. This is a potentially confounding factor because grating resolution varies with the number

of cycles N displayed in the stimulus, even when resolution is sampling-limited.[11,12] Although the number of cycles becomes important only for small N, this condition is not uncommon in peripheral-vision experiments in which conventional video displays may have space for only a few cycles of a coarse, but just-resolvable, grating.

Additional uncontrolled factors in various studies might include individual differences among subjects or variability in the optical refractive error of the peripheral field. Most of the previous experiments on peripheral letter acuity were conducted without correction of peripheral refractive error on the grounds that small amounts of refractive error are not important in peripheral vision.[13–17] However, recent experiments have demonstrated the importance of accurate focusing to maximize the visibility of high spatial frequencies in the periphery, just as in the fovea.[18,19] Indeed, letter acuity in foveal vision is known to be more sensitive to optical defocus than is grating acuity,[7] probably because of the complicated effect of optical blur on the rich Fourier spectra of letters, whereas grating acuity is largely insensitive to refractive error in the periphery.[19] It is possible, therefore, that uncorrected refractive error in the periphery may account for some of the reported deficit in resolution performance for letters relative to gratings.

If the above methodological factors could be brought under control, any residual differences between acuity for letters and for gratings might provide insight into the nature of those visual mechanisms that are responsible for performing spatial resolution tasks in peripheral vision. For example, the task of letter discrimination might be limited by optical aberrations of the eye's imaging system, by insufficient contrast sensitivity of the visual system, by excessive noise in the visual pathways, or by neural undersampling of the visual stimulus. In this paper we use the term sampling-limited to mean that misrepresentation of the stimulus caused by neural undersampling is the primary limiting factor for performance. Over the past decade a large body of evidence has accrued in support of the hypothesis that spatial resolution acuity for extended, high-contrast grating patterns in the peripheral field is sampling-limited and is determined by the sampling density of retinal ganglion cells.[12,20–39] The main lines of evidence for this conclusion are the subjective perception of spatial and motion aliasing, the objective demonstration that detection acuity exceeds resolution acuity, and the close correlation between resolution acuity and the anatomical estimates of ganglion cell density.

Given the success of sampling theory at accounting for peripheral visual acuity of gratings, the most parsimonious model is that the same neural undersampling mechanism also limits letter acuity. We therefore adopted this idea as our working hypothesis in the present study and went on to speculate that the apparent rejection of this hypothesis by previous reports in the literature may have been flawed by a combination of methodological factors and a failure to take into account the importance of the rich spatial Fourier spectra of letters compared with gratings when those authors were formulating a basis for their comparison.

Unfortunately, tests of sampling-limited performance developed previously for gratings do not apply to letter targets because letters are not isoluminant with the background and therefore can be detected as simple luminance increments (or decrements) even when they are minuscule. Therefore to test our working hypothesis we took a more indirect route of measuring peripheral acuity for three types of visual stimulus that bridged the gap from extended gratings, which are known to be sampling-limited, to conventional letter optotypes, while controlling the methodological factors identified above. In this transition from gratings to letters we looked for signs that the critical spatial-frequency components of the target were near the Nyquist frequency of the retinal mosaic when the acuity end point was reached. When this happened we interpreted the result as evidence that target discrimination is sampling-limited. The difficulty with this approach is that we do not have absolute prior knowledge of which spatial frequency components of letters are critical for performing the task. To overcome this deficit we developed the concept of a difference spectrum introduced by Anderson[40] as a tool for identifying reasonable candidates. The rationale behind the difference spectrum is that Fourier components that are common to both letters cannot be used to discriminate those letters. By calculating the difference between Fourier spectra of two letters, we isolate those components that are different in the two stimuli and therefore might be useful for discriminating them.

Our results support the sampling hypothesis for those targets that most resembled a patch of sinusoidal grating (i.e., three-bar targets or a block-E letter with long strokes). However, the other letter pairs tested (i.e., block-E letters with short middle strokes or mirror-symmetric letters E) failed to provide evidence that letter discrimination is sampling-limited. This negative result does not necessarily rule out the sampling hypothesis, however, because our method for identifying the critical frequency components for discrimination may be flawed. Therefore additional tests of the sampling hypothesis were devised based on a paradigm of spatial filtering of letters, the results of which are reported in a companion paper.[41]

2. EXPERIMENTAL RATIONALE

According to the sampling theory of visual resolution, the acuity limit is reached when the stimulus becomes too fine to be represented faithfully by the discrete array of retinal neurons, thus producing spatial aliasing and misperception of the target's features.[39] This theory is comprehensive enough to apply to other stimuli besides high-contrast gratings, provided that the physical attributes of the stimuli are taken into account. For example, theory predicts that peripheral acuity for low-contrast gratings will remain sampling-limited, as opposed to filtering-limited or noise-limited, as long as target contrast is sufficient to exceed threshold for detection. Sampling theory thus accounts for two major ways in which peripheral vision is qualitatively different from foveal vision: (1) peripheral grating acuity is independent of contrast over a large range,[35] and (2) it is independent of optical blur over a large range.[18,19]

The sampling theory of visual resolution has also been used to explain the effect of window size on grating resolution in the periphery.[12] Those grating experiments of Anderson *et al.* were the starting point for the present study designed to follow a staged transition from the known (grating acuity) to the unknown (letter acuity). When extended gratings were viewed through a window that exposed only a portion of the stimulus, resolution acuity fell systematically with the number of visible cycles. Grating resolution remained sampling-limited for truncated gratings (provided that at least one full cycle of the grating was displayed), as demonstrated by the subjective appearance of aliasing and by the objective finding that detection acuity exceeded resolution acuity. This was a surprising result because in any simple model of neural sampling the Nyquist frequency would be a fixed quantity set by the anatomical density of the neural array and therefore should be independent of stimulus parameters. To account for their paradoxical experimental results, Anderson *et al.* analyzed the spatial-frequency spectrum of windowed gratings and found that a large fraction of the stimulus energy was dispersed by truncation into the aliasing zone beyond the Nyquist frequency. An example of this behavior for the more germane and slightly more complicated example of a square-wave grating is illustrated in Fig. 1. Figure 1A shows the contrast spectrum of an extended square-wave grating ($N = 32$ cycles), and Fig. 1B shows the spectrum of the same grating viewed through a square window only 2.5 cycles in width. (For simplicity of presentation, these spectra were computed for one-dimensional cross sections by use of trigonometrical basis functions.) In both cases the characteristic frequency of this stimulus (i.e., the fundamental harmonic component of the square wave) is shown lying just below the Nyquist frequency of the neural sampling array, which is assumed to be a fixed barrier set by the anatomical density of retinal neurons. According to the sampling theory of resolution, the extended grating of Fig. 1A is just resolvable because the critical component, in this case the grating's characteristic frequency, is adequately sampled and therefore will be represented veridically within the visual system. However, when the same grating is truncated, as shown in Fig. 1B, the stimulus is no longer resolvable because a large fraction of the energy in the characteristic frequency has been dispersed into the aliasing zone.

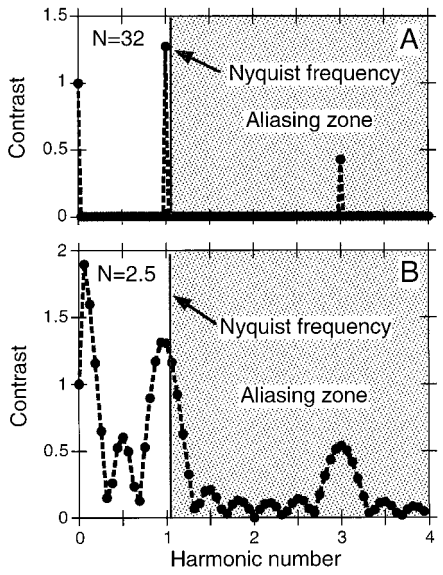

Fig. 1. Contrast spectra for patches of a square-wave grating containing A, 32 stimulus cycles or B, 2.5 stimulus cycles. Both targets are assumed to have the same physical spatial frequency, which lies just below the Nyquist frequency of a neural sampling array. Spectral dispersion causes stimulus energy in the fundamental Fourier component in B to leak into the aliasing zone, whereas no such leakage occurs in A.

The dispersive effect of stimulus truncation in the presence of neural undersampling has at least two deleterious consequences. First, undersampling reduces the amount of veridical energy available for performing the resolution task. This happens when truncation shifts the stimulus energy to frequencies beyond the Nyquist limit of the neural array. These supra-Nyquist components will be represented nonveridically by the neural image, and therefore the amount of veridical signal energy available to support visual resolution must be reduced and may be insufficient to perform the task. We call this account of spatial resolution as a sampling-limited task the energy insufficiency hypothesis. The second consequence of neural undersampling is that the portion of the stimulus energy that lies in the aliasing zone of spatial frequencies beyond the Nyquist limit may actively mask the detection of the remaining veridical energy and for this reason hinder performance. We call this account of spatial resolution as a sampling-limited task the masking hypothesis. The masking hypothesis is supported by previous experiments that showed that supra-Nyquist stimulus energy has a powerful masking effect on the detection of the sub-Nyquist components of compound gratings in peripheral vision.[37] Because the dual effects of reduced veridical energy and active masking by nonveridical energy are both consequences of undersampling, we use the overarching term "sampling-limited" to mean that either, or both, of these two effects is the main factor that limits performance on a visual task.

It follows from the above reasoning that to avoid the deleterious effects of undersampling that hinder resolution of truncated gratings one must reduce the grating frequency such that the stimulus energy will withdraw behind the Nyquist barrier. In other words, acuity will fall as window size shrinks, even when the Nyquist frequency is fixed and performance is sampling-limited. This loss of acuity that is due to truncation can be substantial. Anderson *et al.* found that peripheral acuity for a patch of N cycles of a sinusoidal grating is reduced by the factor $N/(N + 1)$ relative to acuity for an extended grating.[12] From this empirical finding they computed that[37] patches of sinusoidal gratings are not resolvable if a significant portion (e.g., >5%) of stimulus energy in the characteristic frequency is forced by truncation to exceed the Nyquist limit.[12]

Although the dual hypotheses of energy insufficiency and alias masking are conceptually distinct, they are confounded by the traditional acuity paradigm in which a visual target is reduced in size until it is just resolvable. As the target shrinks, the critical frequency component of the stimulus crosses the Nyquist boundary, causing a de-

crease in veridical energy and a simultaneous increase in masking energy. Naturally we would like to know whether one of these two factors is more important than the other. A previous finding[35] that peripheral grating acuity is largely independent of stimulus contrast over a large range (10–100%) would suggest that high-contrast stimuli have a large energy reserve, which would make them robust to the transfer of energy from veridical to nonveridical forms. However, those results were obtained for extended gratings and may not apply to truncated gratings or letters. Furthermore, we suspect that the absolute amount of stimulus contrast may be less important than the relative balance between veridical and nonveridical stimulus energy. This opinion is based on our previous finding that the acuity end point for patches of sinusoidal grating is reached when this balance reaches a critical ratio (e.g., 95:5), regardless of the number of stimulus cycles in the grating.[12]

The preceding analysis of the acuity paradigm was cast in the framework of the sampling theory of resolution, but we could have performed a similar analysis for a filtering-limited system, e.g., foveal vision. In that case the cutoff frequency of the visual filter would have replaced the Nyquist limit of Fig. 1 as the barrier to resolution. The main difference between these two models is that the aliasing zone would now become a zone of invisibility. This comparison between foveal and peripheral vision helps to clarify the essential feature of sampling-limited performance, as distinct from filtering-limited performance. If neural sampling is the limiting factor, then aliased stimulus energy remains visible but either is useless for performing the given psychophysical task (because the energy is nonveridical) or is an active hindrance (because of masking). For these reasons we concluded previously that peripheral resolution of truncated, high-contrast, sinusoidal gratings is a sampling-limited task, provided that the target is reasonably well focused[19] and contains at least one full cycle of the grating.[12] That conclusion was based on the subjective observation of perceptual aliasing of truncated gratings and the objective demonstration that subjects are able to detect patches of grating that were too fine to be resolved. This superiority of detection acuity over resolution acuity is widely considered the definitive test for sampling-limited performance[23,25,42] but is valid only if one removes luminance cues by ensuring that the mean luminance of the target equals that of the background. Unfortunately, the latter condition is not fulfilled by ordinary letters, which is why we adopted the indirect approach of the present study.

3. METHODS

A. Stimuli

The three visual targets used in this study are illustrated in Fig. 2: a three-bar grating, a block-E letter in which all strokes have the same length, and a block-E letter with a short middle stroke. These stimuli were chosen for study because they represent a systematic transition from a truncated square-wave grating to an example of an uppercase, san-serif letter. Taking stroke width as a unit of measure, all targets were 5 units wide and 5 units tall, with 1-unit spacing between strokes. Accordingly,

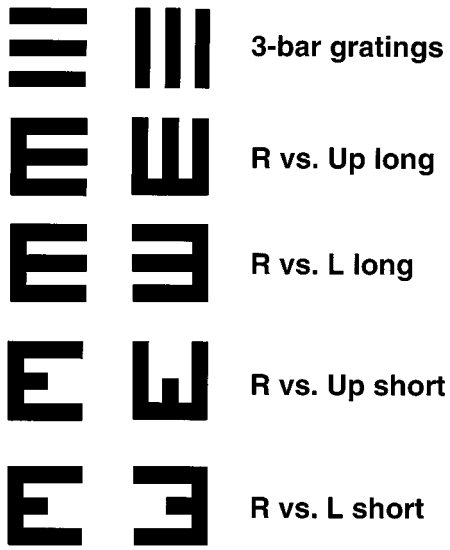

Fig. 2. Five stimulus pairs used for measuring discrimination acuity.

threshold letter size θ refers to both the height and the width of the target. The three-bar stimulus is the U.S. Air Force three-bar resolution test target, which has also been incorporated into some clinical ophthalmic instruments.[43] The long-stroke E is the traditional tumbling-E stimulus used clinically, and we formed the short-stroke E from the long-stroke E by halving the length of the middle stroke. Stimuli were displayed on a monochrome computer monitor (Radius, Inc.) as black targets on a white background of luminance 90 cd/m^2 (contrast, 84%). Further details of the stimulus parameters are available in an earlier paper.[12] For the purposes of this paper we refer to all these targets as letters.

The object frequency of the characteristic component of all of the stimuli used in this study was $f_C = 2.5$ c/let. Dimensional analysis indicates that the physical frequency of this characteristic component at any given letter size θ can be computed with the formula

$$\frac{f_c \text{ cycles}}{\text{degree}} = \frac{f_c \text{ cycles}}{\text{letter}} \frac{1 \text{ letter}}{\theta \text{ arc min}} \frac{60 \text{ arc min}}{\text{degree}}. \quad (1)$$

The same formula can be used to convert units for any other frequency component by dropping the subscript c.

B. Psychophysical Tasks

We used a two-alternative, forced-choice staircase procedure to determine the minimum target size required for discrimination of letter pairs. Five different pairings were used, as illustrated by the five rows in Fig. 2. In every case the letters differed only in orientation, and thus the method was essentially an orientation-discrimination task. We refer to these orientation combinations as right versus up (R vs. U) and right versus left (R vs. L; see Fig. 2). The rationale for making these particular pairings was to include target comparisons with different degrees of similarity. For example, the Fourier spectra of R vs. U targets are rotated by 90° with respect to each other, which means that their amplitude and phase spectra are both different. However, for R vs. L

targets the Fourier spectra are rotated by 180°, which means that the targets differ only in their phase spectra.

C. Estimation of Neural Nyquist Limit

It was shown in a companion study[12] that resolution acuity for sinusoidal gratings is nearly independent of the number of cycles in the target, provided that there are at least six cycles present. Those results provided a psychophysical estimate of the Nyquist frequency set by the spatial density of the neural mosaic at select locations in the peripheral field for the same observers used here. In addition, resolution acuity for interference fringes, which is also known to be a sampling-limited task, was available for these subjects from a separate study.[32] To convert the Nyquist frequency f_N expressed in physical units to object frequency for comparison with the spectra of letters at visual threshold, we used the relationship

$$\frac{f_N(\text{c/let})}{f_N(\text{c/deg})} = \frac{f_c \text{ c/let}}{f_c \text{ c/deg}}. \tag{2}$$

D. Experimental Procedure

Two experienced observers (the authors, RSA and LNT) served as subjects. Their right eyes were refracted by an experienced optometrist who used retinoscopy at eccentricities out to 50° in the temporal visual field (nasal retina). The appropriate correction was then placed in front of the eye in line with the peripheral target but avoiding the central line of sight. For low eccentricities the correcting lens could not avoid the foveal line of sight for the fixation target, in which case the distance to the fixation target was adjusted to avoid stimulating accommodation. The refractive corrections for each subject at each eccentricity are given in Table 1.

The subject sat with his head supported by a chin rest and carefully fixated an illuminated cross at a distance of 3 m, with eyes in the primary position of gaze. Natural pupils were used throughout (4–5 mm for both subjects). Viewing distance to the stimulus was 3 m at each eccentricity except 0°, where we placed the monitor at 7 m to obtain stimuli of sufficiently small angular subtense. For each stimulus presentation the subject indicated the orientation of the target by pressing one of two buttons on a computer keyboard. Viewing time was unlimited to allow for focused attention and critical inspection of the target. Occasionally a subject would experience fading of the peripheral target (Troxler effect), in which case he was encouraged to look off to the opposite side from the monitor for a few seconds before resuming the experiment. The staircase paradigm used a three-down, one-up rule with a step size of 10%. The first two reversals of the staircase were discarded, and the threshold was determined as the average of the last seven reversal values. Two threshold measurements were made for each of the five stimulus pairs at each eccentricity.

Table 1. Refractive Error at Several Eccentricities for both Subjects

	Subject	
Eccentricity	RSA	LNT
0	+0.50 DS	−0.75/−0.50 × 90
5	+0.50 DS	−0.50/−0.50 × 90
10	+0.50 DS	−0.25/−0.75 × 90
20	+1.25/−0.75 × 90	+0.25/−1.25 × 90
30	+2.25/−2.25 × 90	+0.50/−1.75 × 90
40	+3.25/−3.50 × 90	+1.00/−2.25 × 90
50	+4.00/−4.50 × 90	+2.50/−3.50 × 90

To judge the functional significance of any differences measured for the various conditions of our experiments we adopted the conventional clinical criterion that a 25% change in the minimum resolvable letter size is highly significant. This corresponds to a change of 0.1 log unit, which is one full line on a standard Bailey–Lovie eye chart.[44] A fractional change of one letter, or 0.02 log unit, is the smallest change that one would normally expect to measure reliably in foveal vision, but such a small change would usually be considered only marginally significant. Although clinical experience is not as rich for assessing peripheral visual acuity, we adopted these same standards for central and peripheral vision in the present study.

4. RESULTS

The variation of resolution acuity with eccentricity for the long-stroke E and the three-bar grating is compared in Fig. 3. At 20° eccentricity subject RSA found that the stimulus was too close to the blind spot to permit him make reliable judgments, so this eccentricity was omitted for that subject. Error bars representing ±1 standard deviation of the two threshold measurements were typically smaller than the diameter of the symbols shown in Figs. 3 and 4. The average ratio of standard deviation/mean for all the data pairs collected in this study was 0.05. The standard deviation of the seven reversals for any given staircase was typically approximately 12% of the mean, which would indicate a standard error of $0.12/\sqrt{7} = 0.045$. When expressed in logarithmic terms, both of these results indicate that the standard deviation of acuity measurements in these experiments corresponds to 0.02 log unit, which is equivalent to a single letter on any given line of a standard Bailey–Lovie eye chart.[44] Since 0.02 log unit is the smallest change in letter acuity that has clinical or functional significance, we conclude from this analysis that the experimental variability in our data was small enough to reveal the smallest acuity change of functional significance.

Consider now the acuity measurements for an extended sinusoidal grating compared with the three-bar letter, which is the same stimulus as $N = 2.5$ cycles of a square-wave grating. Acuity values are plotted on a logarithmic scale, so the statistical variability and the smallest functional differences are of equal size anywhere on the graph. We anticipated that acuity would be worse for the three-bar letter because of truncation of the square-wave grating. We were able to verify this prediction for one test location in the peripheral field by referring to data collected previously on the same subjects in a related series of experiments.[12] Acuity measured in that study for extended gratings located 30° in the periphery for subject RSA was 6.2 c/deg. That study also showed that peripheral-resolution acuity for sine-wave gratings trun-

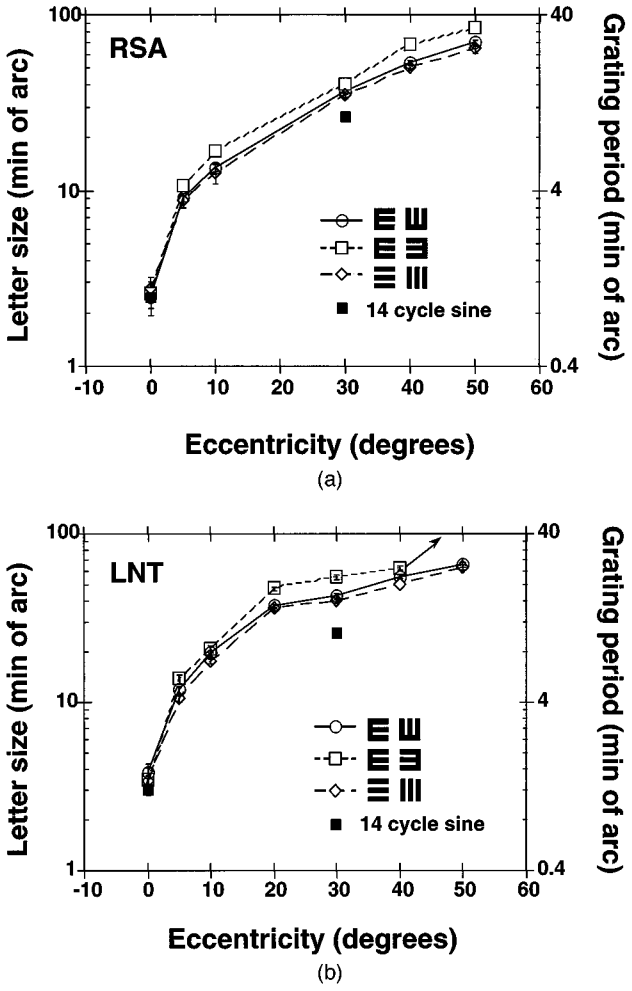

Fig. 3. Threshold letter size and grating period versus eccentricity for long-stroke letters E and three-bar gratings. The labels RSA and LNT refer to the observers and in Fig. 4.

cated to N cycles is reduced by the factor $N/(N + 1)$ relative to acuity for an extended sinusoidal grating. Accordingly, the predicted acuity limit for the three-bar grating used in the present study was 4.4 c/deg, which corresponds to an expected threshold target size of 34 arc min. The corresponding measurement shown in Fig. 3 for subject RSA was 35 arc min, which is not significantly different from the predicted value. The loss of acuity as a result of grating truncation was slightly more than expected for subject LNT (predicted threshold target size, 32 arc min; measured value reported in Fig. 3(b), 39 arc min), but this 0.08-log-unit discrepancy is still less than one line on a Bailey–Lovie eye chart. In summary, peripheral acuity for the three-bar letter is significantly less than acuity for an extended sinusoidal grating but is nearly the same as expected for a patch of sinusoidal grating containing 2.5 cycles, which is known to be a sampling-limited task. Thus we conclude that resolution of the three-bar letter is a sampling-limited task. These results further suggest that the presence of higher harmonics of the square wave have little or no effect on the resolution task, which is consistent with previous experiments that have shown that the higher harmonics of a square wave are not visible in the periphery when the fundamental component is just resolvable.[29,37]

Next consider the results shown in Fig. 3 for the R vs. U discrimination of the long-stroke letter E. Under the conditions of our experiments, acuity for this letter discrimination task is nearly identical to acuity for three-bar letters at every eccentricity tested. The average differences between the two data sets were 0.01 log unit (RSA) and 0.03 log unit (LNT), neither of which is large enough to be judged functionally significant. In other words, adding the fourth stroke to convert a three-bar pattern into a long-stroke E has no effect on orientation discrimination of these targets in the R vs. U configuration. Thus we conclude that discrimination of R vs. U orientations of the long-stoke E letter is also a sampling-limited task, just as we found above for the three-bar letter.

The final data set shown in Fig. 3 is for discrimination of R vs. L orientations of the long-stroke letters E. Acuity for this task was significantly worse than for three-bar gratings at all test locations outside the fovea. The average differences between the two data sets were 0.07 log unit (RSA) and 0.05 log unit (LNT), which are both functionally and statistically significant. Of course the discrimination of R vs. L orientations of a three-bar grating is impossible, so the mere fact that the task could be performed for letters (except for target eccentricities beyond 40° for subject LNT) is evidence that important, new spatial-frequency components are created when a fourth

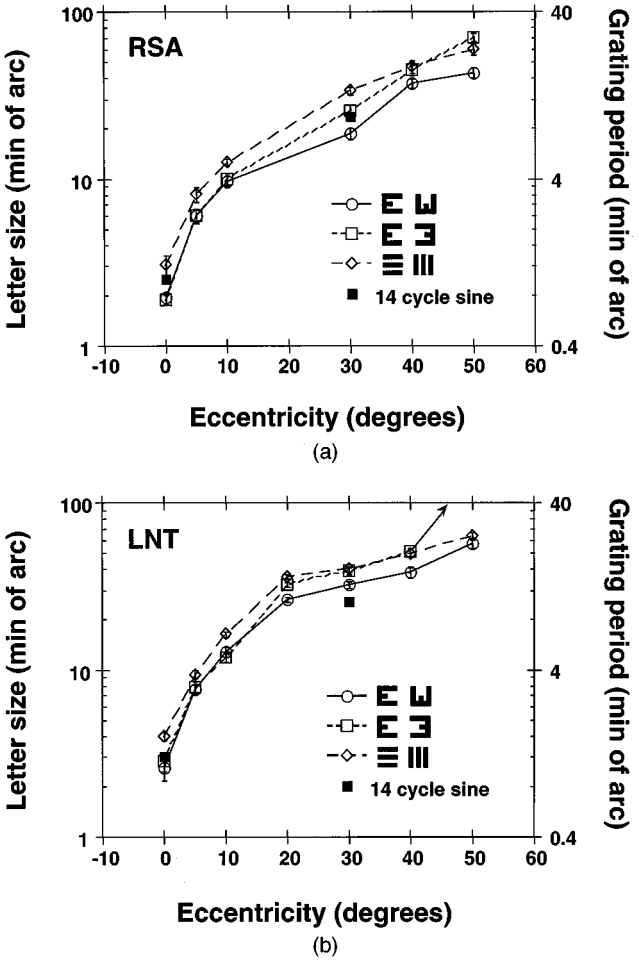

Fig. 4. Threshold letter size and grating period versus eccentricity for short-stroke letters E and three-bar gratings.

Table 2. Foveal MAR (in arc min) and Linear Regression Parameters[a]

	Type of Letter				
	Long E		Three-Bar	Short E	
Parameter	R vs. U	R vs. L	R vs. U	R vs. U	R vs. L
Subject RSA					
MAR	0.49	0.51	0.58	0.39	0.38
b	0.52	0.36	0.62	0.53	−1.07
m	0.54	0.63	0.40	0.43	0.7
*E*2	0.98	0.58	1.55	1.26	−1.53
r	0.99	0.99	0.99	0.98	0.97
Subject LNT					
MAR	0.76	0.69	0.74	0.52	0.57
b	1.87	2.00	1.53	1.08	0.85
m	0.32	0.45	0.31	0.40	0.44
*E*2	5.83	4.46	4.87	2.73	1.93
r	0.99	0.98	0.99	0.99	0.99

[a] MAR = $m \times$ eccentricity + b; $E2 = b/m$; r is the correlation coefficient.

stroke converts a three-bar pattern into a long-stroke letter E. If successful performance of the task is to be attributed to these new spectral components, these unknown critical components must be identified before we can test our working hypothesis that undersampling of them is the liming factor for discriminating the R vs. L long-stroke E's. These issues are reserved for Section 5 below.

Although the peripheral retina is known to be anisotropic, with superior acuity for gratings oriented parallel to visual meridians compared with gratings orthogonal to meridians,[45–47] we do not believe that this feature of the human visual system can explain the differences between R vs. U and R vs. L discriminations documented in Fig. 3. Because the latter stimuli are in the preferred orientation (strokes of E are radially oriented), radial bias would predict that acuity for the R vs. L targets should have been better than for R vs. U, but just the opposite occurred.

Acuity values for the short-stroke letters E are shown in Fig. 4, along with data for the three-bar letter for reference. Surprisingly, acuity for R vs. U short-stroke letters E is significantly *better* than for three-bar letters at all eccentricities tested. The average acuity differences for these two letter pairs were 0.16 log unit (RSA) and 0.11 log unit (LNT), both of which are highly significant functionally and statistically. Although not as striking, a significant difference was also found between acuity for R vs. L orientations of the short-stroke E's. The average differences in this case were 0.07 log unit (RSA) and 0.05 log unit (LNT). In other words, by shortening the middle stroke of the letter E it becomes possible to reduce the size of the letters below the threshold size for three-bar letters and still discriminate their orientation. This suggests that shortening the middle stroke alters the letter's Fourier spectrum in such a way that a different set of spatial frequencies becomes available for performing the discrimination task. Again, we must identify these critical spatial-frequency components before we can determine whether their misrepresentation by neural undersampling is the mechanism that limits the discrimination task.

Figures 3 and 4 employ logarithmic ordinates so error bars are roughly the same size everywhere in them.[12,48] However, we also performed linear regression of the data to determine the value of E2, the eccentricity for which performance doubles with respect to the foveal value.[49] Regression parameters and fovea values of minimum angle of resolution (MAR) are shown in Table 2 for our two subjects. Significant differences between subjects and between letter targets are evident in these data.

5. DISCUSSION

Our results confirm previous reports in the literature that letter acuity in peripheral vision is worse than acuity for extended gratings when the comparison is based on characteristic frequency, which in turn is based on an equivalence between stroke width of letters and half-period of gratings. This difference persists even when the methodological weaknesses of previous experiments are corrected. However, conventional letters are not the only resolution targets for which acuity is worse than for extended gratings. Acuity for a standard three-bar resolution target is also worse than acuity for extended gratings in central or peripheral vision. This is an important result because the three-bar target is a transition stimulus that blends the features of spatially extended sinusoidal gratings (with harmonically pure spectra) and spatially compact letters (with rich Fourier spectra). Therefore, if reduced resolution of the three-bar letter can be attributed to its compact size in conjunction with the neural undersampling mechanism that is known to exist in peripheral vision, then perhaps a similar explanation will apply to the other letters used in this study, and perhaps for alphanumeric characters in general. Because of this pivotal role of the three-bar target in linking visual acuity for letters and gratings, we examine its spectrum in detail next to illustrate our model of the acuity task.

As illustrated in Fig. 1, truncation of an extended square-wave grating to produce the three-bar letter will cause dispersion of each discrete harmonic component of the grating into a band of frequencies centered on the original harmonic frequency. Energy shifted to higher frequencies is useless in an acuity experiment because the acuity end point is determined by the lowest spatial frequency that is sufficient for performing the task.[50,51] Having extra energy at higher object frequencies (i.e., cycles per letter) is never helpful in an acuity experiment because it is the nature of the acuity paradigm that the target be reduced in size until the target is no longer resolvable. As the target shrinks, its spectrum expands, so higher-frequency components cross the cutoff frequency first. The last frequency components to survive are the lowest ones that are sufficient for performing the task. Consequently the end point of an acuity experiment will be determined by that band of frequencies with the lowest number of cycles per letter that is sufficient for performing the task.

Truncation of an extended grating also has the counterintuitive potential for improving acuity. This prediction

follows from the fact that truncation also shifts some of the harmonic energy to lower frequencies, which might make it possible to shrink the target even further and still perform the task. However, two other mitigating factors tend to counteract this advantageous effect of truncation. First, the components dispersed to lower frequencies have lower contrast than the original harmonic, and therefore they might fall below visual thresholds for detection. Second, if the cutoff frequency for resolution is due to a neural sampling mechanism, the detection of sub-Nyquist components can be masked by the energy that was dispersed into the supra-Nyquist domain.[37] We do not know which of these two mechanisms may be more important, but together they evidently outweigh the benefit of spectral dispersion of the characteristic frequency into lower portions of the frequency spectrum. This conclusion is based on previous studies that showed that truncation of a sinusoidal grating reduces, not increases, acuity in both central[52] and peripheral[12] vision. A quantitative analysis of spectral dispersion further leads to an expected loss of acuity as the result of truncation by the factor $N/(N+1)$, as described previously.[12] The present empirical finding that truncation of square-wave gratings reduces acuity (Fig. 3) by the same factor adds further weight to the conclusion that truncation has predictable effects on the Fourier spectrum of the stimulus and on visual discrimination of a given pair of stimuli.

In summary, our interpretation of peripheral resolution of the three-bar letter is as follows: To compensate for the deleterious effects of truncation of a square wave, the period of the three-bar target must be enlarged slightly compared with that of an extended grating so that its spectrum will shrink, thus returning most of the stimulus energy to the veridical zone below the Nyquist sampling limit.[12] Although this increase in target size is interpreted as a reduction in acuity, the underlying mechanism of neural sampling by a fixed array presents the same barrier to resolution of truncated gratings as it does to extended gratings. In other words, differences in acuity for extended and truncated gratings explained by differences in the stimulus without postulating changes in visual mechanism. This suggests that differences in acuity for discriminating other letter pairs might also be explained in a similar way by a model of neural sampling of harmonically rich visual stimuli. We pursue this suggestion next.

A. Two-Dimensional Sampling-Theory Model of Letter Discrimination

Full development of a sampling model of letter discrimination requires a two-dimensional description of the Fourier spectra of the targets. For example, power spectra of the long- and short-stroke E targets in the standard orientation are shown in Fig. 5. (Unlike for Fig. 1, here we used complex exponential basis functions to compute the discrete Fourier transforms of the targets, which accounts for the display of positive and negative frequencies.) Unlike the harmonically pure spectrum of an extended sinusoidal grating, stimulus energy in these letter targets is dispersed over a broad spectrum of frequencies and conditions. The key question to be answered is this: Which

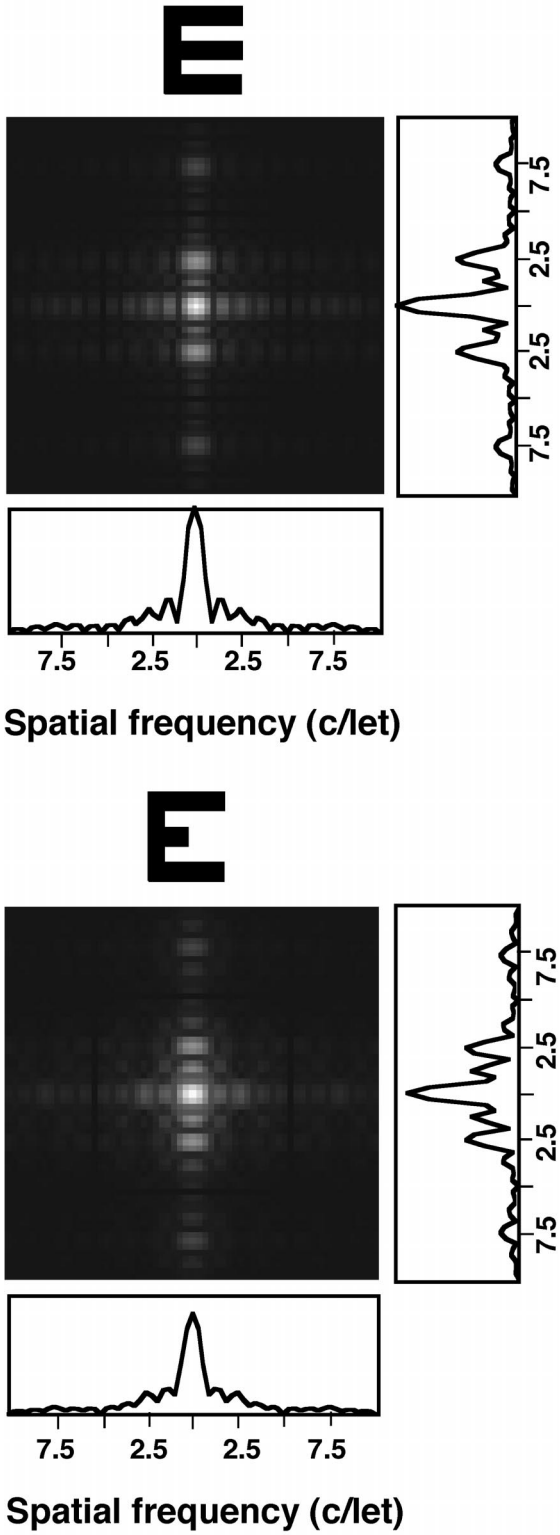

Fig. 5. Contrast spectra for the long- and short-stroke tumbling E's. Spectral resolution in this and other figures is 0.3125 c/let. In the two-dimensional spectra the abscissa and the ordinate represent spatial frequencies in the horizontal and the vertical directions, respectively, and contrast of individual frequency components is encoded by the intensity of the corresponding pixels. One-dimensional spectra below and to the side are cross sections of the two-dimensional spectra taken through the origin. The characteristic frequency in both examples is 2.5 c/let (ordinate), and a prominent second peak occurs also at 1.25 c/let.

spatial-frequency components are used by observers to discriminate targets when the targets are at the minimum discriminable size?

One obvious candidate is the component with maximum contrast, which is at the characteristic frequency (2.5 c/letter) for all the targets used here. If this component is the critical feature for discriminating letters, then according to the sampling theory of resolution the end point of an acuity experiment will be achieved when the letter is reduced in size until the characteristic frequency just begins to exceed the fixed Nyquist spatial frequency of the neural sampling array. If this were the primary factor of importance, performance should have been the same for all our experiments, because all the stimuli had the same characteristic frequency. Since this prediction was not confirmed experimentally, undersampling of the characteristic frequency cannot be the common limiting factor for all of the discrimination tasks performed in our study.

In short, although a simple analysis based solely on the characteristic frequency is sufficient to account for acuity for extended and truncated gratings, it cannot account for the variability of performance at discriminating various letter pairs, all of the same characteristic frequency. One possible explanation for this is that other frequency components besides the characteristic frequency may be responsible for performance in the acuity task. An attempt to identify these additional components is described in Subsection 5.B.

B. Analysis of Difference Spectra

As an aid to identifying the lowest-spatial-frequency components that are sufficient to permit target discrimination, we adopted the approach introduced by Campbell and Robson.[50] They found that a sine-wave grating could be discriminated from a square-wave grating of the same spatial frequency and the same fundamental contrast if the third harmonic in the square wave was at or above its own visual threshold. We can understand this result by considering the difference between the Fourier spectra of the two targets. The difference spectrum isolated those harmonic components that distinguish the two targets and therefore could form a basis for their visual discrimination. According to our sampling-theory model of the acuity task, the end point of an acuity experiment is reached when the lowest spatial frequency that is sufficient for the two targets to be discriminated reaches the Nyquist limit. Presumably this crucial, low-frequency component is present in the difference spectrum, and our problem is to identify it. For example, in the Campbell–Robson experiment the third harmonic is the lowest-frequency component of the difference spectrum that is sufficient for discriminating the targets. Thus the difference-spectrum method of analysis successfully identified the visibility of the third-harmonic component as the key factor in an acuity experiment for discriminating sine-wave gratings from square-wave gratings. More recently, the difference-spectrum method was developed further for evaluating acuity for uppercase letters[40] and for Landolt-C targets[51] as well as for investigating the crowding phenomenon.[53] In the present study we used the difference image and the difference spectrum as tools for specifying how pairs of stimuli differ. Of course there are similarities as well that might influence performance,[54] but the first step is to find out what is different about two targets that makes them potentially discriminable.

An example of the difference-spectrum method is shown in Fig. 6 for the three-bar letter used in our experiments. In computing this difference spectrum we took phase as well as contrast into account by treating the Fourier spectrum as a complex-valued quantity. Because the Fourier transform is a linear operation, the difference of Fourier spectra of two targets is equal to the Fourier spectrum of the difference between the targets. Thus the simplest way to compute the difference spectrum was to subtract the intensity values of one target from those of the other on a pixel-by-pixel basis as shown in Fig. 6 and then compute the Fourier spectrum of this difference image. The result is a spectral description of the difference between the two targets that takes account of both phase and contrast differences between targets and helps us to visualize those spatial-frequency components that are different in the two targets and therefore could form a basis for their discrimination.

Visual inspection of Fig. 6 reveals a prominent component in the difference spectrum at 2.5 c/let, the characteristic frequency. To compare this prominent frequency with the estimated Nyquist frequency, we used Eq. (1) to convert the coordinates of Fig. 6 from object spatial frequency (cycles per letter) to visual units (cycles per

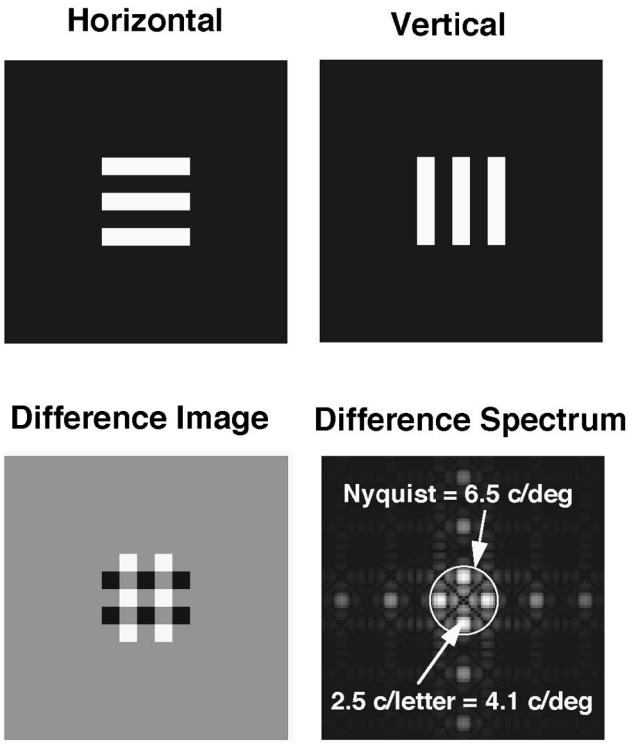

Fig. 6. Determination of the difference spectrum. The images of the horizontal and vertical three-bar gratings are subtracted in the spatial domain to give the difference image. A fast Fourier transform is then performed on the difference image, and the magnitude of the result is displayed as the difference spectrum. Calibration in physical units is for a target at the resolution threshold (34-arc-min target width) for subject RSA, 30° eccentricity.

degree) by taking into account the target size when the letters were just discriminable. For example, at 30° of eccentricity the minimum resolvable target size for a three-bar grating was 35 arc min for subject RSA, which corresponds to a visual frequency of 4.3 c/deg for the characteristic frequency at the acuity end point. Given this absolute calibration of the frequency axis, we superimposed on Fig. 6 a circle of radius of 6.2 c/deg, which we call the Nyquist ring. The radius of this ring represents the estimated Nyquist frequency determined previously for observer RSA at the same retinal locus.[12] A more accurate depiction of the Nyquist limit would stretch the circular ring into an ellipse to take account of the spatial anisotropy of the retina[45–47] and make it a fuzzy border to account for irregularity in the sampling mosaic,[23,37] but those refinements were not implemented here.

Notice that when the target is at the threshold of discriminability, the prominent component identified by the lower arrow in Fig. 6 lies just inside the Nyquist ring. This suggests that veridical representation of the most prominent component of the difference spectrum is required for discriminating the horizontal and vertical orientations of the three-bar target. According to the quantitative theory developed previously[12] and summarized above, the characteristic frequency of this critical component (4.3 c/deg) cannot get any closer to the Nyquist frequency (6.2 c/deg) without dispersing significant amounts of energy (e.g., >5% for the tested peripheral locus) into the aliasing zone that lies outside the Nyquist ring, thereby preventing resolution of the target. This energy dispersion provokes the dual effects of insufficient energy in the remaining veridical components and masking by aliased components, both of which result from neural undersampling. Regardless of which of these two mechanisms may be more important for limiting resolution, the results are consistent with the concept of sampling-limited performance as defined in this paper. In summary, the analysis of Fig. 6 suggests that acuity for orientation discrimination of the three-bar letter in peripheral vision is limited by neural undersampling of those critical Fourier components of the target revealed by the difference spectrum.

To test our working hypothesis that the same neural undersampling model can also account for discrimination performance for tumbling-E letters, we repeated the same graphic analysis described above for Fig. 6 for the other letter pairs shown in Fig. 2. The results are displayed in Fig. 7. In each panel the lower arrow points to the component of the difference spectrum with the highest energy. These spectra were individually calibrated in physical units according to Eq. (1) to permit a direct comparison of the neural Nyquist limit with the various frequency components in the stimulus when the letters are just discriminable. Of these four examples, the case of R vs. U long-stroke E's (Fig. 7a) is most similar to the case of the three-bar letter described above. The component of the difference spectrum with highest energy is at the characteristic frequency (2.5 c/letter), and thus the predictions given above for three-bar letters also pertain to the acuity limit for this letter pair. In this case the minimum resolvable letter size was 37 arc min, for which the characteristic frequency was 4.1 c/deg, which is close

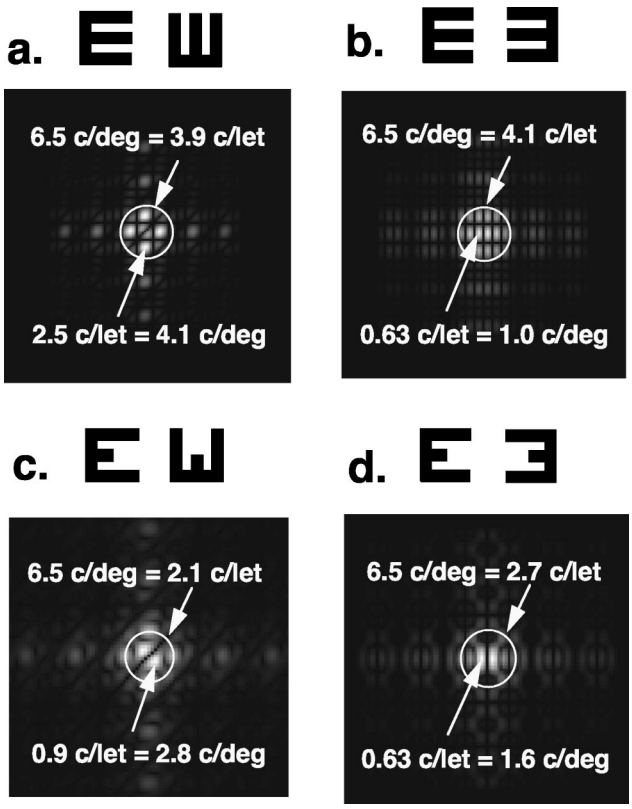

Fig. 7. Difference spectra for test targets when letter size is at psychophysical threshold for discrimination (subject RSA, 30° eccentricity). The Nyquist frequency estimated from a previous paper[12] is shown by a circle centered on the origin. Notice that the most prominent components of the difference spectrum (marked by arrows) vary with the letter pair being discriminated but in every case fall inside the Nyquist ring. Spatial-frequency calibration is provided in terms of object frequency (cycles per letter) and physical frequency (cycles per degree).

(0.03-log-unit discrepancy) to the predicted value of 4.4 c/deg and therefore confirms the predictions of the sampling model.

Analysis of the other three pairs of letters shown in Fig. 7 indicates that the spatial-frequency components with the most energy in the difference spectra have spatial frequencies less than half the characteristic frequency. Why this should be true is most easily understood for the R vs. L target pairs. For example, the only difference between the R and L long-stroke E's shown in Fig. 7b is in the position of the vertical stroke, which is represented in the frequency domain mainly by the phase of the low-frequency vertical components. Accordingly, the most prominent component of the spectrum in Fig. 7b has the frequency 0.63 c/let on the abscissa, which corresponds to a physical frequency of 0.9 c/deg at the acuity end point. Based on this assessment of the difference image and the difference spectrum, three arguments lead us to conclude that discrimination of this mirror-symmetric pair of letters is *not* sampling-limited. First, the discrepancy between an acuity of 0.9 c/deg and the Nyquist limit of 6.2 c/deg is too great to be accounted for by experimental variance. Second, the available evidence obtained with truncated sinusoidal gratings indicates that at least one complete cycle must be present in the stimulus for visual resolution to be sampling-limited.[12] Third, inspec-

tion of the difference image for R vs. L E's suggests that the discrimination task requires a position judgment of the vertical stroke, which is a difficult task in the periphery because of the steep falloff of hyperacuity in peripheral vision and its dependence on the quality of the reference structure.[55] Thus we have no reason to expect this particular discrimination task to be sampling-limited, and our evidence confirms this expectation.

On the basis of the preceding analysis, we must reject the hypothesis that undersampling of the most prominent frequency component in the difference spectrum is the mechanism that limits discrimination of R vs. L long-stroke E's or discrimination of either configuration of the short-stroke E's. This is not a fully satisfying conclusion, however, because it is possible that discrimination of these particular letter pairs is determined by undersampling of a critical component in the difference spectrum that has less energy but higher frequency than the most prominent component. If so, then performance may still be limited by neural undersampling but for some less-conspicuous frequency component. The spectra of Figs. 7b–7d suggest several possible candidates for the critical component, which are investigated further in the companion study.[41]

C. Importance of Low Spatial Frequencies

The difference spectra displayed in Figs. 6 and 7 indicate the presence of various amounts of signal energy below the characteristic frequency of 2.5 c/let. Might these low-frequency components be the critical components for letter discrimination? Clearly, if neural undersampling is the mechanism that limits discrimination, then low frequencies that are well below the Nyquist frequency (even when the letter is slightly smaller than the acuity limit) cannot be the critical components that limit acuity. However, it is possible that some other mechanism is responsible for the end point of acuity, which could reinstate the importance of the low frequencies. For example, letter discrimination might fail when the low-frequency components fall below the threshold of visual detection mechanisms. In the following discussion it is important to bear in mind that the present study does not address the discrimination of letters that are larger than the acuity limit. Our only concern here is to explain why a particular pair of letters can be discriminated when they are at the acuity limit but become indiscriminable when reduced further in size.

To examine the importance of low-frequency components in the difference spectrum, the same two observers participated in another study that systematically removed the low-frequency components of letters by high-pass filtering. If the low-frequency components are critical for letter discrimination, then acuity should suffer when they are removed by filtering. The results of that series of experiments, described in the companion paper,[41] show that removing the frequency components below the prominent band of frequencies near the characteristic frequency has little or no effect on acuity for discriminating the R vs. U long-stroke E pair. This result supports the conclusion drawn above that this particular discrimination task is sampling-limited. To the contrary, results for filtered R vs. U short-stroke E's point to the prominent band of oblique frequencies at ~1 c/let in the difference spectrum as necessary and sufficient for achieving normal levels of acuity. Since this frequency band is well below the Nyquist frequency at the acuity end point, those results support the conclusion drawn above that discrimination of R vs. U short-stroke E's is not a sampling-limited task. By the same logic, the discrimination of R vs. L letter pairs (short or long stroke) is not sampling-limited. Discrimination of these mirror-symmetric letters is much less robust to high-pass filtering, which indicates the need for low-frequency components well below the Nyquist frequency for normal acuity to be achieved.

Taken together, these results suggest that as letters depart from the simple structure of a truncated square-wave grating, thereby developing a richer Fourier spectrum, frequency components other than the characteristic frequency play an increasingly important role in the visual task of letter discrimination.[51] Performance in any given discrimination task thus depends on the particular letters that are being discriminated and on the specific way in which their Fourier spectra differ. For this reason, easy generalizations across all letter combinations may not be possible. Nevertheless, it seems worth reconsidering the energy integration model developed previously to help to weigh the balance between the veridical energy in the difference spectrum that falls below the Nyquist frequency and the aliasing energy above. Such a model provided a heuristic account of empirical measurements of the variation of acuity with number of cycles in a patch of sinusoidal grating across a range of retinal eccentricities.[12] To see whether a similar model might also account for letter acuity, we computed the total power inside and outside the Nyquist ring in each of the difference spectra shown in Figs. 6 and 7. We found that in every case approximately 80% of the stimulus energy in the difference spectrum lay inside the Nyquist ring when the targets were just resolvable. Although this finding may be mere coincidence, it suggests that letter discrimination in a sampling-limited domain may be determined by a balancing of the amount of stimulus energy inside the Nyquist ring (which provides beneficial, veridical information) against energy outside the Nyquist ring (which provides nonveridical, aliased information). Thus in a descending-staircase paradigm the target size may be reduced in size to the point where a significant amount of energy (e.g., 20%) leaks out of the Nyquist ring, leaving insufficient veridical information with which to perform the task. The end point for letter discrimination would therefore be determined jointly by the neural Nyquist frequency and the precise spectral distribution of energy in the particular letters that are being discriminated.

D. Limitations of Difference-Spectrum Analysis

We emphasize that the difference-image–difference-spectrum method described above for identifying components of a visual target that may support visual discrimination is based solely on the properties of the stimulus and takes no account of the properties of the human visual system other than the neural sampling density. Our approach was to minimize the number of assumptions made about the nature of visual processing in the

peripheral field, choosing instead to examine how far we can go toward accounting for acuity simply on the basis of the physical stimulus and the anatomical fact that the optical image is spatially sampled by the neural retina. Although our computation of the difference spectrum takes full account of contrast and phase information in the targets, the resulting spectrum may differ in detail from the information that is actually used by the peripheral visual system to perform a task. In particular, our analysis takes no account of the reduced sensitivity to contrast[28,35,56–58] or to spatial phase[4,17,59–62] recorded in the literature of peripheral vision. Here we are interested only in identifying those object components that are potentially useful for letter discrimination. It would take additional work to determine whether the contrast or the phase (i.e., position) of these components is more important for target discrimination. In fact, our use of the difference-spectrum method to analyze letter pairs is compatible with either phase or contrast discrimination models. Similarly, the sampling model that is tested in this study applies equally well to phase discrimination and to contrast discrimination mechanisms since both stimulus attributes are veridical below the Nyquist frequency and nonveridical above. Consequently our account of the acuity task is necessarily incomplete, and we presume that a physiologically based, sampling-limited model with such features as impoverished phase or contrast sensitivity, contrast adaptation, probability summation, localization, masking, optical aberrations, and other visual factors will provide a better explanation of the experimental results. Nevertheless, we believe that, it is important first to understand the nature of visual stimuli, before and after neural sampling, before inferring the nature of visual processing from measures of visual performance.

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